

Mid-year Exam Study Guide

Tables:

1. Find the closed form definition of the functions that fits these tables:

a.

x	f(x)
0	4
1	7
2	10
3	13
4	16

$f(x) = 3x+4$

b.

x	f(x)
0	12
1	10
2	8
3	6
4	4

$f(x) = -2x+12$

c.

x	f(x)
0	2
1	6
2	14
3	26

x	f(x)
4	42

$$f(x) = 2x^2 + 2x + 2$$

d.

x	f(x)
0	15
1	22
2	36
3	57
4	85

$$f(x) = 3.5x^2 + 3.5x + 15$$

2. Fill in the following table using the function:

a.

x	f(x)
0	-20
1	-15
2	-10
3	-5

$$f(x) = 5x - 20$$

b.

x	f(x)
0	16
2	-12
6	-140
10	-364

$$f(x) = 3x^2 - 8x + 16$$

3. Use Lagrange interpolation to find a function that fits the table.

a.

x	f(x)
1	3
2	7
4	21

$$f(x) = 1(x-2)(x-4) - 3.5(x-1)(x-4) + 3.5(x-1)(x-2)$$

OR

$$f(x) = x^2 + x + 1$$

b.

x	f(x)
1	4
3	40
5	156

$$f(x) = 0.5x(x-3)(x-5) - 10(x-1)(x-5) + 19.5(x-1)(x-3)$$

OR

$$f(x) = 10x^2 - 22x + 11$$

c.

x	f(x)
1	6
2	15
3	36

$$f(x) = 3(x-2)(x-3) - 15(x-1)(x-3) + 18(x-1)(x-2)$$

OR

$$f(x) = 6x^2 - 9x + 9$$

Errors:

1. Calculate mean absolute error, mean squared error, and standard error.

a. $y = 4x - 7$

x	f(x)
1	-10
2	8
3	12
4	2

Mean Absolute: 7

Mean Squared: 49

Mean Absolute: 7

b. $y = 4x - 19$

x	f(x)
4	-10
5	8
6	12
7	2

Mean Absolute: 7

Mean Squared: 49

Mean Absolute: 7

c. $y = 12x - 5$

x	f(x)
1	-22
2	58
3	40
4	24

Mean Absolute: 24

Mean Squared: 701

Mean Absolute: 26.48

d. $y = 12x - 29$

x	f(x)
3	-22
4	58
5	40
6	24

Mean Absolute: 24

Mean Squared: 701

Mean Absolute: 26.48

Recursive monthly payments:

1. For each of the following problems, create the recursive function that models amount owed after each month and create a table for the first three months of payments.

a. You take out a \$10,000 loan to buy a car at 5% annual interest and make \$300 monthly payments.

$$f(x) = \begin{cases} 10,000 & \text{if } x=0 \\ f(x-1) + f(x-1)(.05/12) - 300 & \text{if } x>0 \end{cases}$$

Month 3: \$9221.76

b. You buy a car for \$15,000 and make a down-payment of \$8,000. You take a loan for the rest at 4.5% annual interest and make \$250 monthly payments.

$$f(x) = \begin{cases} 7,000 & \text{if } x=0 \\ f(x-1) + f(x-1)(.045/12) - 250 & \text{if } x>0 \end{cases}$$

Month 3: \$6326.23

c. You buy a car for \$5000 and take out a loan for 50% of the value at 3.8% annual interest with \$150 monthly payments.

$$f(x) = \begin{cases} 2,500 & \text{if } x=0 \\ f(x-1) + f(x-1)(.038/12) - 150 & \text{if } x>0 \end{cases}$$

Month 3: \$2072.40

Factorial Functions:

1. Find the value of each expression.

a. $12!$

479,001,600

b. $2! + 8!$

40,322

c. $\frac{14!}{5!}$

362,880

2. Solve each equation for n

a. $\frac{n!}{(n-2)!} = 90$

n=10

b. $\frac{(n+1)!}{(n-1)!} = 72$

n=8

c. $\frac{(n+1)!}{n!} = 36$

n=35

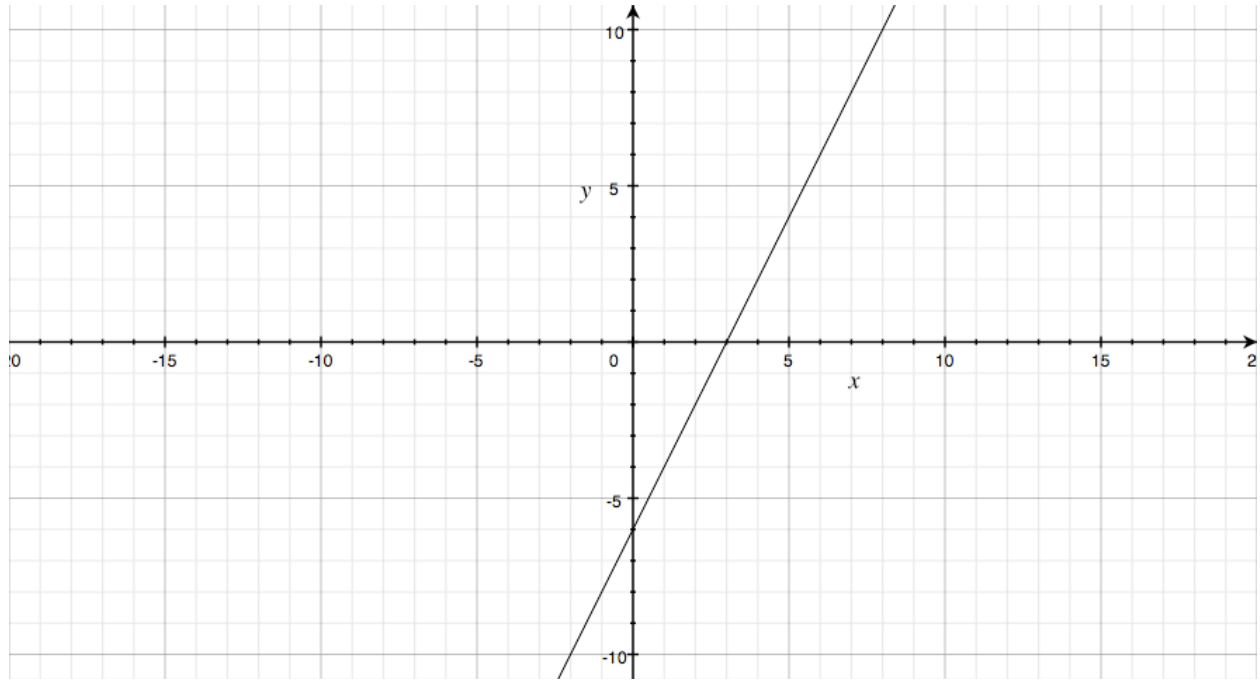
d. $\frac{(n-1)!}{(n-3)!} = 56$

n=9

Domain and Range:

1. Find the domain and range of each function.

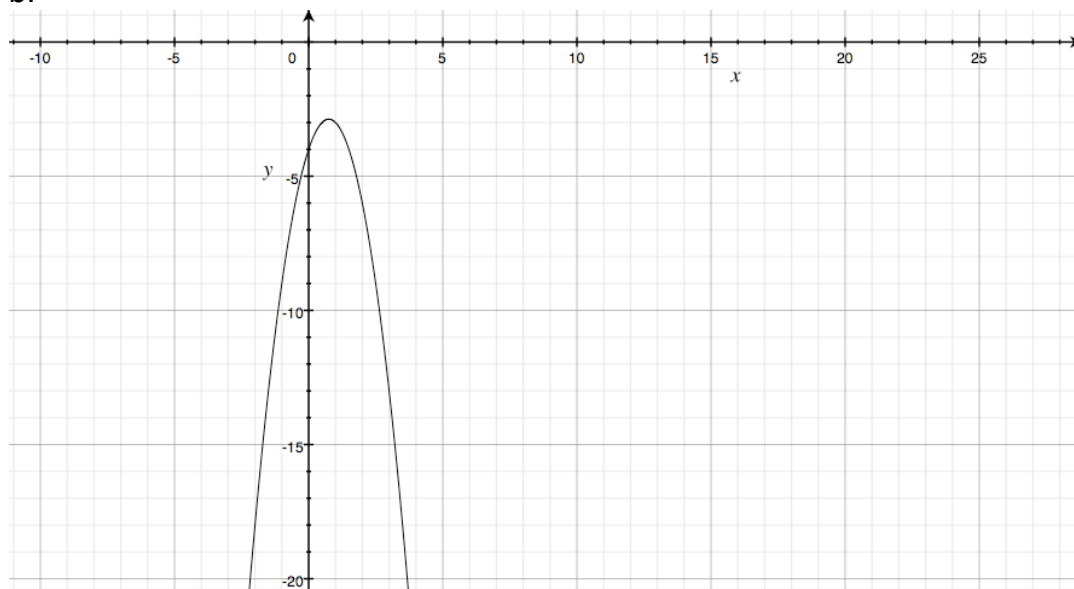
a.



$$D: \infty < x < \infty$$

$$R: \infty < y < \infty$$

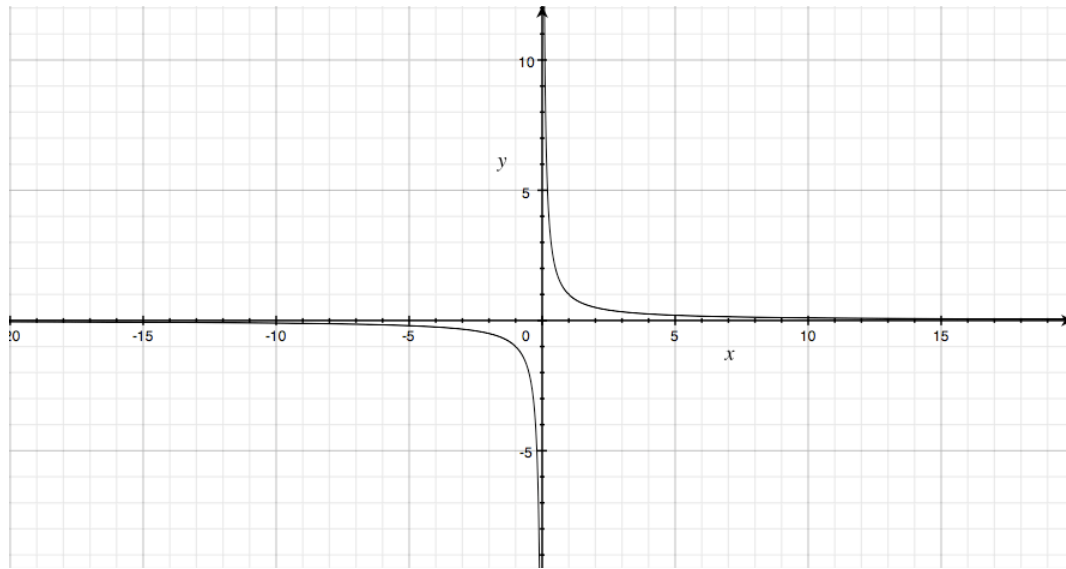
b.



$$D: \infty < x < \infty$$

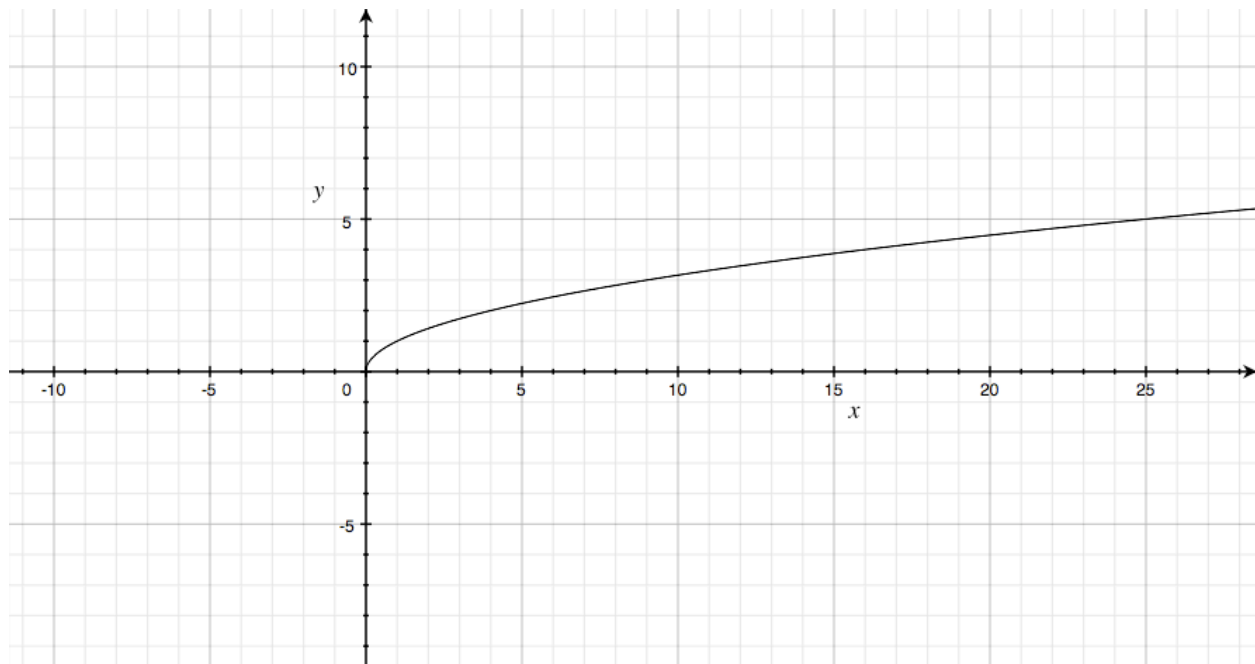
$$R: \infty < y \leq -3$$

c.



D: $-\infty < x < \infty, x \neq 0$
R: $-\infty < y < \infty, x \neq 0$

d.



D: $0 \leq x \leq 28$
R: $0 \leq y \leq 5$

Composing functions:

1. Use the definition of functions f and g to find each value.

$$f(x) = 4x - 6$$

$$g(x) = x^2 + 3x$$

a. $f(10) = \mathbf{34}$

b. $f(g(6)) = \mathbf{210}$

c. $g(f(2)) = \mathbf{10}$

d. $f(g(x)) = \mathbf{4(x^2 + 3x) - 6}$

2. Use the definition of functions f and g to find each value.

$$f(x) = (x - 2)^2$$

$$g(x) = 2x + 7$$

a. $f \circ g$

$$\mathbf{(2x + 5)^2}$$

b. $g \circ f$

$$\mathbf{2(x - 2)^2 + 7}$$

c. $f \circ g(5)$

$$\mathbf{225}$$

d. $g \circ (f \circ g)$

$$\mathbf{2(2x + 5)^2 + 7}$$

Inverses:

1. Function f is one to one and $f(6) = 10$ and $f(12) = 16$.

a. $f^{-1}(10) = \mathbf{6}$

b. $f^{-1}(16) = \mathbf{12}$

2. Find the inverse of each function.

a. $f(x) = 3x + 5$

$$f^{-1}(x) = \frac{x - 5}{3}$$

b. $f(x) = 3x - 11$

$$f^{-1}(x) = \frac{x + 11}{3}$$

c. $f(x) = -10x + 4$

$f^{-1}(x) = \frac{-x-4}{10}$

d. $f(x) = \frac{x}{3} = 12$

$f^{-1}(x) = 3(x+12) = 3x+36$